Multifractal Detrended Cross-Correlation Analysis of Agricultural Futures Markets

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Abstract: We investigated geographically far but temporally correlated China's and US agricultural futures markets. We found that there exists a power-law cross-correlation between them, and that multifractal features are significant in all the markets. It is very interesting that the geographically far markets show strong cross-correlations and share much of their multifractal structure. Furthermore, we found that for all the agricultural futures markets in our studies, the cross-correlation exponent is less than the averaged generalized Hurst exponents (GHE) when $q < 0$ and greater than the averaged GHE when $q > 0$.

Key words: Agricultural futures market; Cross-correlation exponent; Multifractal spectrum width

1 Introduction

It is a great challenge for econophysicists to identify the underlying patterns of spatial and temporal complexities in social and economic systems. In current literature, few results could be found to incorporate geographically far but temporally correlated economic or financial systems and treat them as one system with interacted components. For example, although many scientists attempted to uncover the patterns and their formation mechanisms in China's and/or US agricultural futures markets, they failed to realize that many of the markets are actually geographically and/or temporally correlated, so that they merely discussed these markets separately instead of treating them as interacting subsystems in one integrated global or regional market.

Empirical evidence supports the existence of fractals or multifractals in either China's or US agricultural futures markets. For instances, Corazza *et al.* studied six main US agricultural futures markets and found the existence of fractals [1]; Chatrath et al. studied four futures markets as the representatives of US agricultural futures market and found low-dimensional chaotic structures [2]; similar results show that China's agricultural futures market is also multifractal [3, 4]. Therefore, a single fractal dimension can not explain the scaling geometry of the market patterns, which may be better described by a spectrum of scaling exponents. Although empirical studies are found, the previous researches analyzed market dynamics separately then simply compared the phenomena obtained from the isolated analyses, ignorance of the complex cross-correlations between those geographically and temporally correlated markets.

Many scholars have studied the cross-correlation among financial time series, such as stocks [5-9] and currencies [5], using the methods which assume that both of analyzed time series are stationary. But unfortunately, in the real world, especially in the case of economic and financial markets, the time series are usually nonstationary; therefore, the approaches in current literature may lead to a spurious detection of auto- and cross-correlation. Therefore, many scientists incorporated the temporal and/or spatial factors and applied Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to investigate the cross-correlation between two signals, which may successfully overcome this limitation in current literature. For example, Jun *et al.* proposed a detrended cross-correlation approach to quantify the correlations between positive and negative fluctuations in a single time series [6]. Based on the previous studies, Podobnik and Stanley proposed Detrended Cross-Correlation Analysis (DCCA) to investigate power-law cross-correlations between two simultaneously recorded time series in the presence of nonstationarity [7]. Podobnik *et al.* then applied their new method to uncover long-range power-law cross-correlations in the random part of the underlying stochastic process [8] and the cross-correlation between volume change and price change [9]. To unveil multifractal features of two cross-correlated nonstationary signals, Zhou proposed Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to combine Multifractal Detrended Fluctuation Analysis (MF-DFA) and DCCA approaches [10].

In this paper, we applied MF-DCCA to analyze the geographically correlated agricultural commodity markets, i.e. hard winter wheat, soy meal, soybean and corn, in China and US. We arrive at the following nontrivial conclusions: firstly, there exists a power-law cross-correlation between these futures markets; secondly, all of cross-correlation relationships are also found to be multifractal; thirdly, the relationships are investigated between cross-correlation exponent and the generalized Hurst exponent in single markets; finally, the price change of integrated soy meal market is a random walk process than those of single markets both in China and US.

2 Method

Let us briefly introduce MF-DCCA method [10, 11]. Suppose that there are two time series $x(i)$ and $y(i)$ $(i = 1, 2, \cdots, L)$, where L is the length of series. For each series, we calculated the absolute logarithmic returns:

$$|R_x(i)| = \left|\log \frac{x(i+\Delta t)}{x(i)}\right|, \ |R_y(i)| = \left|\log \frac{y(i+\Delta t)}{y(i)}\right|, i = 1, 2, \cdots, L - \Delta t \tag{1}$$

where $\Delta t = 1$. Then the “profile” is determined:

$$X(i) = \sum_{k=1}^{i} [|R_x(k)| - \langle |R_x| \rangle], Y(i) = \sum_{k=1}^{i} [|R_y(k)| - \langle |R_y| \rangle], i = 1, 2, \cdots, L - \Delta t \tag{2}$$

We divided the entire series into $N_s = N - s + 1$ over-lapping boxes with the length of s. And then for each of the N_s segments the local trends are estimated by means of the mth order polynomial fit. Then the detrended covariance is given by

$$F_v(s) = \frac{1}{s} \sum_{k=1}^{s} \left|X_v(k) - \tilde{X}_v(k)\right| \times \left|Y_v(k) - \tilde{Y}_v(k)\right| \tag{3}$$

where $\tilde{X}_v(k)$ and $\tilde{Y}_v(k)$ are the fitting polynomials in segment v respectively. Definitely our definition is different from those in other references[1]. And then the qth order is defined as

$$F_{xy}(q, s) = \left[\frac{1}{N_s} \sum_{v=1}^{N_s} F_v(s)^{\frac{q}{2}}\right]^{\frac{1}{q}} \tag{4}$$

When q=0, the limit of Eq.(4) can be given by

[1] In reference [10] the detrended covariance is given by Eq.(3). But we found that in our results $\ln F_{xy}(q, s)$ is nonlinear when q is negative. Zhou also pointed out in reference [14] that for negative q values, no obvious power-law scaling can be identified for the daily closing prices of DJIA and NASDAQ indexes. Therefore, in this paper, Eq.(3) is defined as a variation of the detrended variance in MF-DFA, that is, $F_v^2(s) = \frac{1}{s}\sum_{k=1}^{s}\left(X_v(k) - \tilde{X}_v(k)\right)^2 = \frac{1}{s}\sum_{k=1}^{s}\left|X_v(k) - \tilde{X}_v(k)\right|^2$.

$$F_{xy}(0,s) = \exp\left[\frac{1}{2N_s}\sum_{v=1}^{N_s}\ln F_v(s)\right] \tag{5}$$

If power-law cross-correlations do exist, the scaling or power-law relationship should satisfy

$$F_{xy}(q,s) \propto s^{h_{xy}(q)} \tag{6}$$

The cross-correlation exponent $h_{xy}(q)$ in Eq.(6) can describe the power-law relationship between two temporally correlated time series. Especially, if the time series x is identical to y, MF-DCCA is equivalent to MF-DFA; and if the $q = 2$, the cross-correlation exponent $h_{xy}(2)$ is equivalent to the well-known Hurst exponent $h(2)$. According to Shadkhoo and Jafari [11], the similar relationship between classical multifractal scaling exponents $\tau_{xy}(q)$ and q can be given by

$$\tau_{xy}(q) = qh_{xy}(q) - 1 \tag{7}$$

If $\tau_{xy}(q)$ is linear with q, the cross-correlation of the correlated series is monofractal; otherwise, it is multifractal. By the means of a Legendre transformation, we can obtain the following relationships

$$\alpha = h_{xy}(q) + qh_{xy}^{'}(q), f_{xy}(\alpha) = q(\alpha - h_{xy}(q)) + 1 \tag{8}$$

The strength of multifractality thereby can be estimated by the width of multifractal spectrum [3], which is given by

$$\Delta\alpha = \alpha_{max} - \alpha_{min} \tag{9}$$

3 Data Analyses and Discussions

3.1 Data

The data used in this paper are the daily closing prices of China's and US representative commodity futures markets (data source: Reuter© Database), that is, hard winter wheat futures prices from Dec. 28th, 1993 to Jun. 3rd, 2010 ($L = 3334$) from China's Zhengzhou Commodity Exchange, and soy meal futures prices from Jul. 17th, 2000 to Jun. 3rd, 2010 ($L = 2369$), No. 1 soybean futures market from Mar. 15th, 2002 to Jun. 3rd, 2010 ($L = 1981$), corn futures market from Sep. 22nd, 2004 to Jun. 3rd, 2010 ($L = 1382$) from China's Dalian Commodity Exchange. Meanwhile, same length of daily closing prices of wheat futures market ($L = 3334$), soy meal futures market ($L = 2369$), soybean futures market ($L = 1981$) and corn futures market ($L = 1382$) are also chosen from Chicago Board of Trade (CBOT) within the similar time span.

To better describe the time series, the integrated profiles $I(n) = \sum_{i=1}^{L-\Delta t}(|R(i)| - \langle|R|\rangle)$ are illustrated in Fig. 1. In our calculation, the order q ranges from -5 to 5 with step 0.1, scale s ranges from 10 to $\left[\frac{L-\Delta t}{10}\right]$, polynomial order $m = 1, 2, 3$ and all of the boxes are overlapped.

All our programming and calculations are performed in Matlab©.

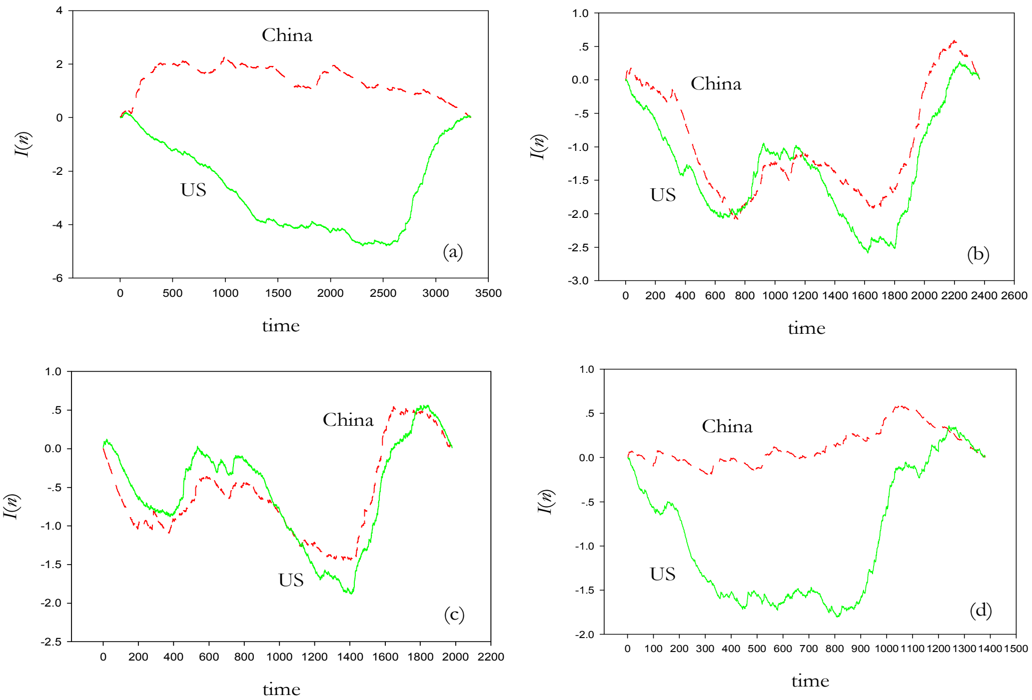

Figure 1. The integrated profiles of the absolute logarithmic returns for both of China's and US agricultural futures markets ((a) hard winter wheat, (b) soy meal, (c) soybean and (d) corn futures markets)

3.2 Results and Discussions

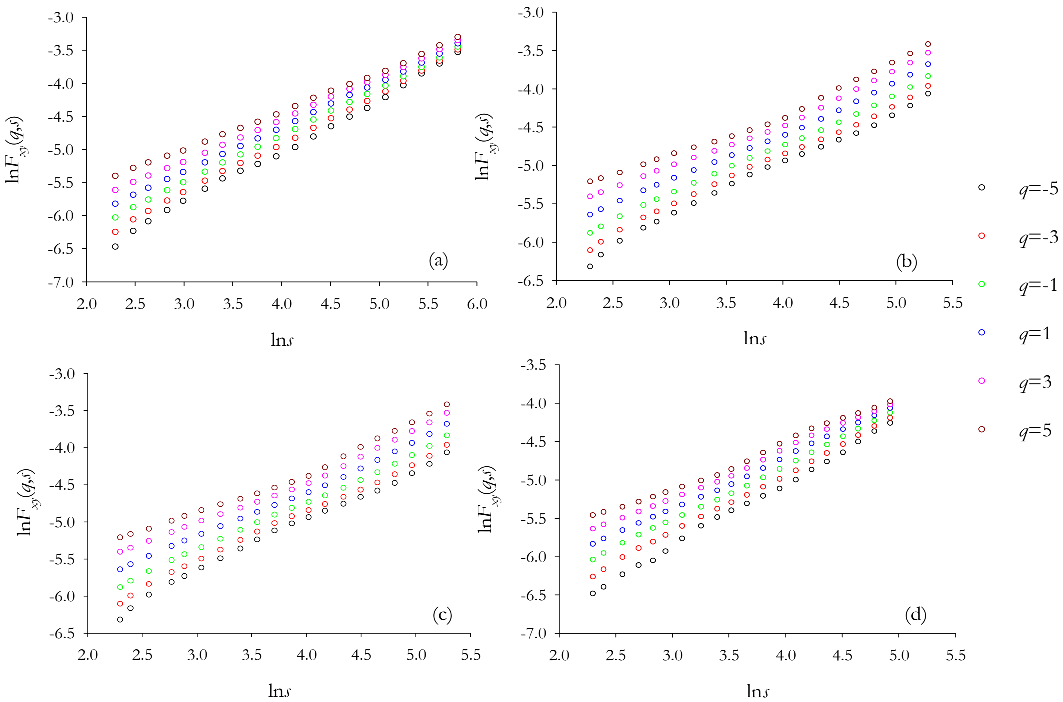

Figure 2. The linear relationship between $\ln F_{xy}(q,s)$ and $\ln s$ for China's and US futures markets when polynomial order $m=3$ ((a) hard winter wheat, (b) soy meal, (c) soybean and (d) corn futures markets)

By means of above-mentioned method, we intend to (i) identify whether there exists a multifractal scaling law in the markets; and (ii) quantify the multifractal strengths of the systems (if any). If multifractality does exist, it may help us identify, explain and control the underlying physical mechanisms that dominate and govern the market dynamics; thus, it would greatly deepen our understanding of market macro phenomena [12].

Fig. 2 shows the log-log plot of $\ln F_{xy}(q, s)$ versus ln$s$ between China's and US futures markets when polynomial order $m = 3$ (when $m = 1$ and 2, the results are qualitatively similar). For different q, all of the curves are linear, which suggests that there exist power-law cross-correlations in each pairs of futures markets. For example, China's Wheat is power-law cross-correlated with US Wheat (see Fig. 2 (a)). The power-law cross-correlation relationship indicates that a large increment of price change in a futures market may be more likely to be followed by a large increment of price change in the other geographically or temporally correlated futures market.

In Fig. 3 the relationship is displayed between cross-correlation exponent $h_{xy}(q)$ and q (black curves). To make a comparison, we also estimated the generalized Hurst Exponent $h(q)$ of each separate analyzed futures market by means of MF-DFA (red curves stand for China's markets, and green curves represent US markets in Fig. 3). If exponent h is a constant, the market is monofractal, otherwise it is multifractal. From this plot we can find that the relationships are multifractal because for different *q*, there are different exponents h; that is, for different q, there are different power-law cross-correlations.

We obtained the cross-correlation exponents and generalized Hurst exponents when $q = 2$ (see Table 2). As we know, Hurst exponent describes the persistence of auto-correlation in a separately analyzed time series. If Hurst exponent $h(2) > 0.5$, the system exhibits persistent properties; if $h(2) < 0.5$, it is anti-persistent. But for the cross-correlation exponent, it only describes the exponent of power-law relationship when $q = 2$. Furthermore, Podobnik and Stanley found, both numerically [7] and analytically [8], that the cross-correlation exponent is equal to the average of individual Hurst exponents for two fractionally autoregressive integrated moving average (ARFIMA) processes sharing the same random noise when $q = 2$. Zhou found that for two time series constructed by binomial measure from *p*-model, there exits the following relationship [10]:

$$h_{xy}(q) = \frac{h_{xx}(q) + h_{yy}(q)}{2} \qquad (10)$$

We calculated the average of generalized Hurst exponents (the pink curves in Fig. 3), which is an average of separately analyzed China's and US markets (the red curves and green curves in Fig. 3). In our results we found that for all of the agricultural futures markets in our studies, the cross-correlation exponent is less than the averaged generalized Hurst exponents (GHE) when $q < 0$ and greater than GHE when $q > 0$.

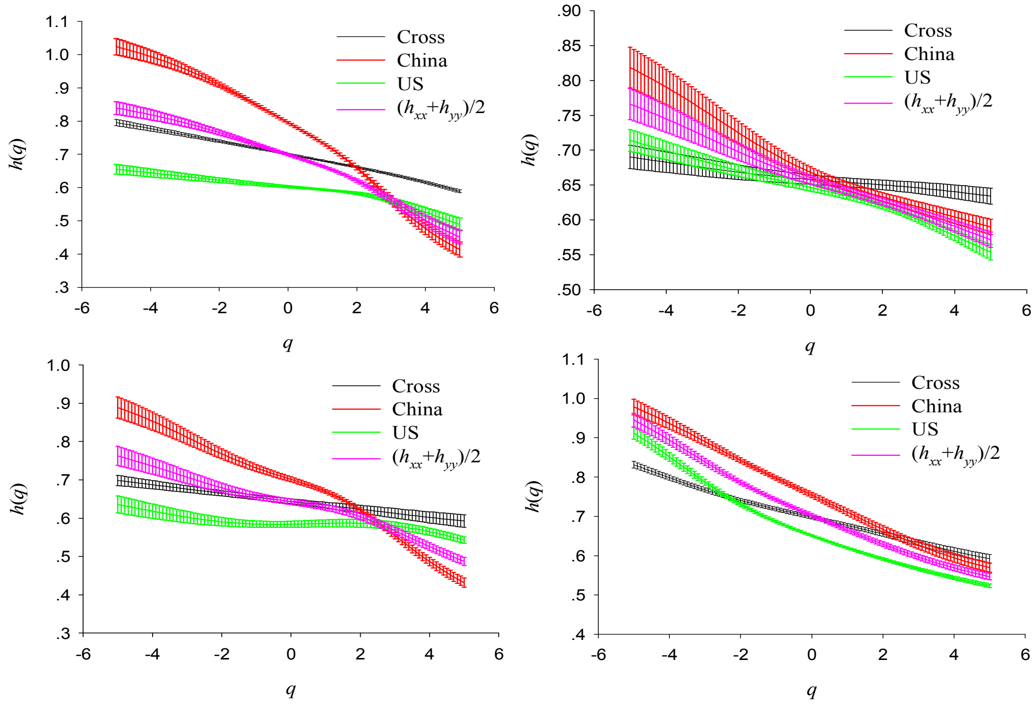

Figure 3. The results of $h(q)$ as a function of q between China's and US agricultural futures markets when polynomial $m = 3$ ((a) hard winter wheat, (b) soy meal, (c) soybean and (d) corn futures markets). The red curves and green curves are obtained by MF-DFA, black curves are calculated by MF-DCCA and the pink curves are the average of General Hurst Exponent $h_{xx}(q)$ (China) and $h_{yy}(q)$ (US).

By means of Eq. (7), the multifractal exponent $\tau_{xy}(q)$ is estimated (see Table 1 and Fig. 4). In Table 1, the slopes of tangents to the two tails of $\tau(q)$ are listed, from which one can obviously find that for all the markets, τ is nonlinearly dependent on q, and this is another piece of evidence of multifractality.

Table 1 The slopes of tangents to the two tails of $\tau(q)$

		$m=1$		$m=2$		$m=3$	
		$q=-5$	$q=5$	$q=-5$	$q=5$	$q=-5$	$q=5$
	China	1.1098	0.3218	1.1364	0.2444	1.1523	0.1566
hard winter	US	0.7689	0.4123	0.7008	0.3321	0.7086	0.3162
	Cross	0.9221	0.4637	0.8891	0.4516	0.8818	0.4508
	China	0.8773	0.6134	0.9267	0.5759	0.9454	0.5210
soy meal	US	0.8170	0.5239	0.7379	0.4654	0.7828	0.4371
	Cross	0.7763	0.6984	0.7002	0.6417	0.7264	0.6021
	China	1.0017	0.3210	0.9841	0.2038	1.0464	0.1934
soybean	US	0.7186	0.4468	0.6816	0.4076	0.7236	0.4251
	Cross	0.7856	0.7596	0.7187	0.6084	0.7526	0.5481
	China	1.0168	0.4915	1.1602	0.4897	1.1665	0.4619
corn	US	1.2712	0.5387	1.1800	0.4256	1.1849	0.4315
	Cross	0.9349	0.6486	1.0220	0.5318	0.9930	0.5025

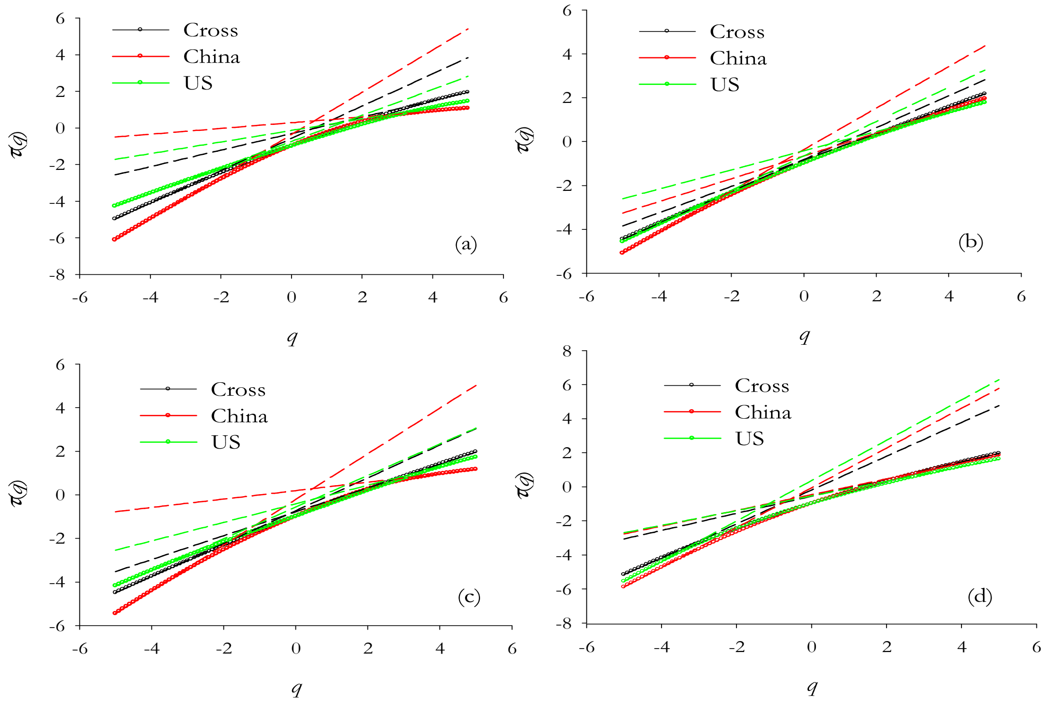

Figure 4. The relationships between $\tau(q)$ and q between China's and US agricultural futures markets when polynomial $m = 3$ ((a) hard winter wheat, (b) soy meal, (c) soybean and (d) corn futures markets).

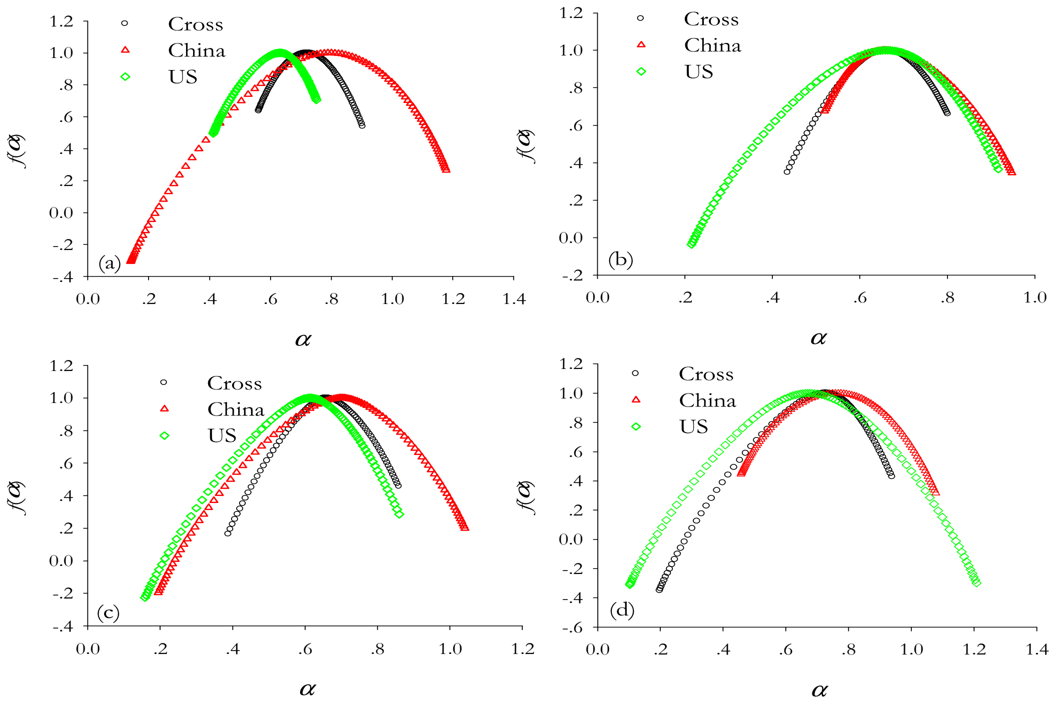

Figure 5 shows the multifractal spectra between China's and US agricultural futures markets when polynomial $m = 3$. Panels (a), (b), (c) and (d) illustrate relationships between $f(\alpha)$ and α in hard winter wheat, soy meal, soybean and corn futures markets respectively.

Then all of the slopes for different q and the multifractal spectra are estimated by means of Eq. (8) and Eq. (9) (see Fig. 4). It is widely known that the multifractal spectrum of monofractality is a point, namely, the width of multifractal spectrum is zero if the system under study is monofractal. Actually the width of multifractal spectrum can be regarded as an estimate of multifractal strength [3]. The numerical results of the widths are listed in Table 1. Especially, for the soy meal, soy bean and corn futures markets, the widths of cross-correlation multifractal spectra are narrower than those of separately analyzed China's and US soy meal futures markets. The widths of cross-correlation multifractal spectra for all markets are significantly nonzero, which imply that there are clear departures from random walk process for either separately analyzed (China's and US) markets, or cross-correlated ones.

Table 2 Hurst Exponents and Multifractal Spectra Widths $\Delta\alpha$

		$h(2)$			$\Delta\alpha$		
		$m=1$	$m=2$	$m=3$	$m=1$	$m=2$	$m=3$
hard winter	China	0.7285	0.6811	0.6590	0.7880	0.8920	0.9957
	US	0.6402	0.5924	0.5828	0.3566	0.3686	0.3924
	Cross	0.7136	0.6785	0.6613	0.4585	0.4375	0.4310
soy meal	China	0.7147	0.6519	0.6329	0.2638	0.3508	0.4244
	US	0.6797	0.6361	0.6193	0.2931	0.2724	0.3457
	Cross	0.7353	0.6718	0.6502	0.0779	0.0585	0.1244
soybean	China	0.7006	0.6370	0.6214	0.6807	0.7803	0.8530
	US	0.6994	0.6164	0.5866	0.2718	0.2740	0.2985
	Cross	0.7506	0.6552	0.6235	0.0586	0.1103	0.2045
corn	China	0.7288	0.6864	0.6670	0.5253	0.6705	0.7046
	US	0.6712	0.5995	0.5916	0.7325	0.7544	0.7534
	Cross	0.7419	0.6698	0.6552	0.2863	0.4902	0.4905

4 Conclusions

In this article, we applied MF-DCCA to investigate the cross-correlation of agricultural futures markets in two geographically far but highly correlated economies, namely, China and USA. Our nontrivial empirical findings can be summarized as follows:

First of all, there exist power law cross-correlations in each pair of futures markets between two countries, which suggests that a large increment of price change in a futures market is more likely to be followed by a large increment of price change in the other related futures market.

Secondly, multifractality is significant in all the cross-correlation relationships of agricultural futures markets in two economies.

Thirdly, we found that for all of the agricultural futures markets in our studies, the cross-correlation exponent is less than the averaged generalized Hurst exponents (GHE) when $q < 0$ and greater than GHE when $q > 0$.

Our results show that although geographically far, China's and US agricultural futures markets show strong cross-correlations and share much of similar multifractal structure.

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